

GUTPA/99/08/3
hep-lat/9908058

Semi-leptonic decays heavy-light to heavy-light

Joachim Hein^{a,*}

^aDept. Physics & Astronomy, University of Glasgow, G12 8QQ, Scotland, UK.

We present results for the QCD matrix elements involved in semi-leptonic decays of B -mesons into pseudo scalar heavy light states. The application of NRQCD heavy quarks allows for quark masses around the physical b -quark. We investigate the dependence of the form factors on the external momenta and looked at the mass dependence at zero recoil. For the first time, results for radially excited decay products are presented.

1. INTRODUCTION

In semi-leptonic decays of B -mesons into D , D^* , D^{**} , D' , ... the CKM-matrix element V_{cb} can be studied cleanly. The extraction of V_{cb} from the experimental data requires the knowledge of the QCD-matrix element $\langle B|V_\mu - A_\mu|D \rangle$. In case of a pseudo-scalar decay product, e.g. D or D' , there is no contribution from the axial current and the matrix element can be described by two form factors F_1 and F_0 resp. h^+ and h^- :

$$\begin{aligned} \langle B|V_\mu|D \rangle &= F_1(q^2) \left[(p_B + p_D)_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu \right] \\ &\quad + F_0(q^2) \frac{m_B^2 - m_D^2}{q^2} q_\mu \end{aligned} \quad (1)$$

$$\begin{aligned} &= \sqrt{m_B m_D} \left[h^+(\omega)(v_B + v_D)_\mu \right. \\ &\quad \left. + h^-(\omega)(v_B - v_D)_\mu \right]. \end{aligned} \quad (2)$$

With $\omega = v_B \cdot v_D$. We use NRQCD to describe the heavy quarks involved. This enables us to use quark masses as heavy as $2m_b$. Hence our calculation covers well the physical b -quark regime. In NRQCD the current $V_0 = 1 + \mathcal{O}(m_Q^{-2})$ is conserved, even on the lattice. Hence there is no current renormalisation in the elastic case [3]. Since NRQCD is cheap in comparison to other lattice techniques, we could apply all our non-zero momenta at source and sink. Furthermore we used 2 different smearings at source and sink. This allowed us to study radially excited states.

*In collaboration with P. Boyle, C.T.H. Davies, Glasgow; J. Shigemitsu, Ohio; J. Sloan, Kentucky. JH, PB and CTHD are members of the UKQCD collaboration.

We used 278 configurations at $\beta = 5.7$ in the quenched approximation provided by the UKQCD collaboration. The lattice volume was $12^3 \times 24$. From am_ρ [1] one determines $a^{-1} = 1.116(12)(^{+56}_{-0})$ GeV. For the heavy quarks we used an NRQCD action up to $\mathcal{O}(m_Q^{-2})$ and also included the $p^4/(8(am_Q)^3)$ term. For details see e.g. [2]. For the mass we used $am_Q = 2.0, 4.0$ and 8.0 . The b -quark corresponds to $am_q = 4.0$. The light quarks were tadpole improved clover with $\kappa = 0.1400$. This corresponds to the strange quark mass as determined from the K -meson. For all am_Q we used a separation of 11 time-slices for the mesonic source and sink. This corresponds to a physical distance of 2 fm. At $am_Q = 4.0$ we also used a separation of 8 time slices. At the source and sink we applied momenta of $\vec{p}^2 \in \{0, 1, 2\}$ in units of $(2\pi/(12a))^2$. This results in $\vec{q}^2 \leq 8(2\pi/(12a))^2$.

In order to isolate the physical states we combined the smearing functions such that we eliminate the ground state or first excited state contribution. The coefficients were determined from matrix fits to meson propagators. Applied to the meson propagator, the excited state smearing delivers a clear signal for up to 5 time slices, before it falls into noise. We apply these improved smearings throughout the calculation.

All reported results are preliminary.

2. ELASTIC SCATTERING

In the elastic equal mass case we obtained the most accurate results with the following ratios,

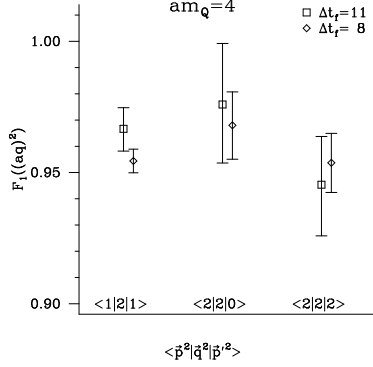


Figure 1. Dependence of $F_1(q^2)$ on the external momentum. The figure gives squared spatial momenta in units of $(2\pi/(aL))^2$.

depending on whether $p^2 = p'^2$ or $p^2 \neq p'^2$

$$\frac{\langle B_1(p) | V_{0,t}(q) | B_{t_f}(p) \rangle}{\langle B_1(p) | V_{0,t}(0) | B_{t_f}(p) \rangle} \quad (3)$$

$$\frac{\langle B_1(p) | V_{0,t}(q) | B_{t_f}(p') \rangle \langle B_1(p') | V_{0,t}(q) | B_{t_f}(p) \rangle}{\langle B_1(p) | V_{0,t}(q) | B_{t_f}(p) \rangle \langle B_1(p') | V_{0,t}(q) | B_{t_f}(p') \rangle} \quad (4)$$

The plateau value of eqn. (3) delivers F_1 the second one $(F_1)^2$. These ratios provide excellent noise cancellation given the non-zero momenta at sink and source that we have.

In figure 1 we investigate the dependence of F_1 on the momentum of the external state. Note the results for $\vec{p}^2 = 2(2\pi/aL)^2$ and $\vec{p}'^2 = 0$ correspond to a very slightly shifted q^2 . The figure shows no significant external momentum dependence. The result is displayed for the two different source and sink separations. No significant dependence on Δt_f arises. This is true for the entire q^2 range we investigated.

In figure 2 we compare the form factors obtained with different am_Q . The results are nicely described by a straight line. We give the following preliminary estimate for the slope of the strange Isgur-Wise function:

$$\rho_{\text{strange}}^2 = 1.5(3)(4). \quad (5)$$

The first parenthesis gives the statistical uncertainty, the second one an estimate of the systematic uncertainties due to residual excitations in

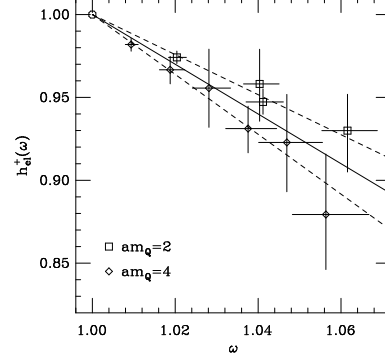


Figure 2. Form factor in elastic scattering. The lines give the slope of the Isgur-Wise function, the dotted ones give the range of it statistical uncertainty.

the ratios, eqn. (3) and (4). The latter is determined from the difference of the two source and sink separations.

3. RADIAL EXCITED STATES

For the first time we have studied semi-leptonic matrix elements to radially excited mesons B' . As described before, we combined our sink smearings such that the overlap with the ground state vanishes. Since we did not observe an excited state signal for more than 5 time-slices, the ratio eqn. (4) was not applicable. The matrix element had to be determined from

$$\frac{\langle B_1(p) | V_{0,t}(q) | B'_{t_f}(p') \rangle}{\langle B_1(p) | B_{t_f}(p) \rangle \langle B'_1(p) | B'_{t_f-t+1}(p') \rangle}. \quad (6)$$

We use the amplitudes determined in double exponential matrix fits to relate the above ratio to the physical matrix element.

With this we achieved a reasonable signal for $am_Q = 4.0$, $\Delta t_f = 8$ and $\vec{p}^2 = \vec{p}'^2 = 2$. For $\vec{q}^2 = 0$ we measure $0.00(2)$ for the matrix element, when fitting 4 and 5 time slices away from the excited state sink. This is compatible with the states being orthogonal. For $\vec{q} \neq 0$ one expects the states to get boosted with respect to each other. Therefore the matrix element should become non-zero. This is shown in figure 3. In order to reduce the statistical noise in the plateau plot, we subtracted

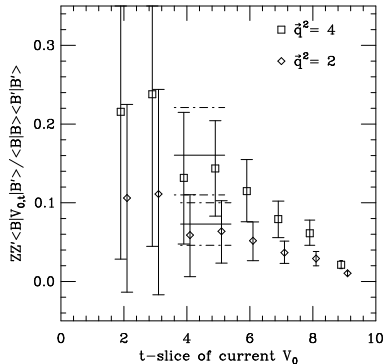


Figure 3. Plateaux of the matrix element of the radial excited to ground state transitions. We applied the radial excited smearing to the sink at $t=9$. The external momentum $\vec{p} = 2\pi/12a$.

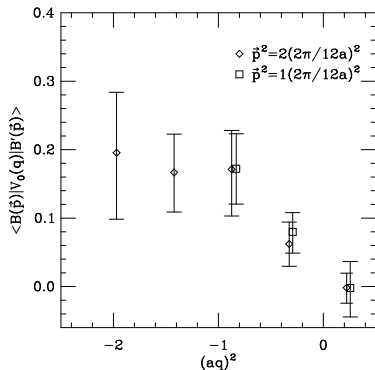


Figure 4. Matrix element for radial excited state for different values of q . The squares are slightly displaced for clarity.

the result for $\vec{q}=0$. This has no significant effect on the fitted values, indicated by horizontal lines. We observe a steady increase of the matrix element with increasing \vec{q} , in agreement with the above expectation.

In figure 4 we summarise the q dependent behaviour of the matrix element. The figure also shows results differing only by the external momenta to agree nicely with each other.

4. NONDEGENERATE TRANSITIONS

At zero recoil we studied the form factor for different values of m_Q at the source and sink. The

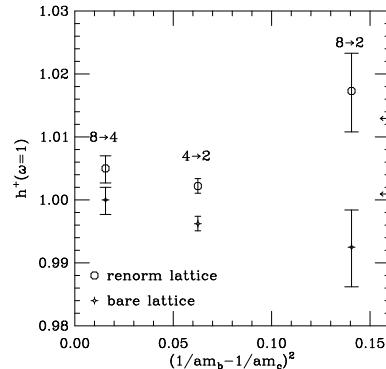


Figure 5. Form factor h^+ at zero recoil

form factor is extracted from the same ratio as in [4]. Due to the different masses the current V_μ is no longer conserved and the form factor requires renormalisation. This has been worked out in one loop perturbation theory in [3]. Our results are displayed in figure 5. We give results for the unrenormalised as well as the renormalised form factor. The static limit to which these results extrapolate to, depends on m_c/m_b . For a comparison, the arrows give the result of [4] for their extrapolation to the $B \rightarrow D$ transition, corresponding to $(1/am_b - 1/am_c)^2 \approx 0.56$.

5. CONCLUSION

We present our first results on semi-leptonic $B \rightarrow D$ decays using realistic values for m_B . We include the elastic scattering case and demonstrate the possibility of studying decays into radially excited states. Our form factors prove to be independent of the momenta of the external states. We give results for the renormalised $h^+(\omega)$ in unequal mass transitions at zero recoil.

The calculations have been performed at NERSC. This research was supported by DOE, the European Commission, NATO and PPARC.

REFERENCES

1. H. Shanahan, et al., Phys. Rev. D55 (1997) 1548.
2. S. Collins, et.al., these proceedings.
3. P. Boyle, C. Davies, these proceedings.
4. S. Hashimoto, et.al., hep-ph/9906376.